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An Interpretation of the "Greatest Hits"-like Behavior

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Abstract

We try to explain the strange behavior for suppliers of books or softwares which release the cheap(paperback) edition of their products only when this product has a good sale, though its cost is nearly zero. We construct a differential equation like the replicator dynamics of a strategic form game, and show that if supplier's decision is biased, then his payoff may be better than not biased.

Keywords: cheap edition, replicator dynamics, biased decision.
JEL classification numbers: C73, D21.

1 Introduction

The purpose of this article is to explain the strange behavior for suppliers of books or softwares which release the cheap(or, paperback) edition of their products only when this product has a good sale[1]. The cost to release the cheap(paperback) edition is usually very cheaper than to create their new product[2], so they can get new profit even if this cheap edition sells only a thausand copies. Therefore, this behavior is strange and seems to contradict

[1]The term "greatest hits" means the series of the cheap edition of the Sony PlayStation's softwares.

[2]So we can assume its cost is approximately 0.

the profit-maximization behavior of suppliers. If they are rational, then they must release the cheap edition of "every" products[3].

This paper tries to explain its behavior and to show that this strange behavior of suppliers does not necessarily mean their irrational choice. The basic idea is here; if they always release the cheap edition, then most consumers wait for the release of cheap edition and tends to not buy it. So that, to control such waiting behavior, supplier does not release the cheap edition when his product does not have a good sale. In other words, suppliers exchange the long-run profit for short-run profit.

To express this idea, we use a dynamic system similar to the replicator dynamics of a strategic form game represents this situation, except the supplier's behavior is biased so that he tends to not release the cheap edition when many consumers wait for it. We show that if this bias is sufficiently high, then supplier's payoff may be improved than his bias is 0.

Compared to other alternative explanations(especially using the folk theorem), this explanation has some virtues. First, this explanation need not require the clever action of consumers. Second, supplier's decision process to realize the long-run equilibrium emerges specifically, and matches actual behavior. So this explanation is better than the others.

In section 2, we display our model and derive the result of this model. Section 3 is the concluding remarks.

2 Model and result

We constracts a strategic form game below;

	R	NR
B	$u - p_H$, p_H	$u - p_H$, $p_H - D$
NB	$d - p_L$, p_L	0, $-D$

and a dynamic system;

$$\dot{x}/x = (1-x)[(u-p_H) - y(d-p_L)],$$

$$\dot{y}/y = (1-y)[(1-x)p_L + D] - b(1-x).$$

[3]Moreover, we can argue that the motivation to release the cheap edition of a product which has a good sale is smaller than which does not have a good sale, because in the latter case this product has missed relatively large potential demand. Although this argument seems to have some persuasive power, it completely contradicts the actual behavior of suppliers.

If $b = 0$, then this system is equal to the replicator dynamics of above game. We assume $p_H > p_L > 0, d - p_L > u - p_H > 0, D > 0$.

The interpretation of these models is here: player 1 represents a consumer and player 2 represents a supplier. Player 1 considers whether he buys a product immediately. If he gets it immediately, then he gains u. If not, he waits for the release of the cheap edition of this product. If player 2 releases the cheap edition, player 1 can get it and gain d. If he does not release it, then player 1 waits until this product becomes old-fashioned and the worth of it tends to 0. p_H denotes the original retail price of this product, and p_L denotes the price of cheap edition. So we assume $p_H > p_L > 0$.

If $u - p_H \geq d - p_L$, then consumer should buy immediately. Such consumer must not relate the determination of release of cheap edition, so we assume $d - p_L > u - p_H$. Moreover, if $u - p_H \leq 0$, then he should wait for cheap edition. Such consumer's behavior is so simple that we treat it only implicitly and assume if player 2 does not release the cheap edition, then he automatically loses $-D < 0$, the sales of such consumer. Therefore, we treat only the case $d - p_L > u - p_H > 0$.

The replicator dynamics of this game represents the short-run behavior of players. At first, both players blindly believe their current action is the best. As time goes by, however, they may observe their action is not the best and change it. Changing action occurs in proportion to the difference between the payoff of current action and of social average. So both the growth rate of the proportion of the people buying immediately(denoted by x) and of the probability of releasing cheap edition(denoted by y) are equal to above difference[4], i.e.,

$$
\begin{aligned}
\dot{x}/x &= u - p_H - [x(u - p_H) - (1 - x)y(d - p_L)] \\
&= (1 - x)[(u - p_H) - y(d - p_L)], \\
\dot{y}/y &= xp_H + (1 - x)p_L - [y(xp_H + (1 - x)p_L) + (1 - y)(xp_H - D)] \\
&= (1 - y)[(1 - x)p_L + D].
\end{aligned}
$$

Finally, the term $-b(1 - x)$ represents supplier's decision is biased in the sense that if x is large and his product does not have a good sale, then he does not release the cheap edition.

Then we get a result below;

proposition 1 (1)If $b \leq \dfrac{(p_L + D)[(d - p_L) - (u - p_H)]}{d - p_L}$, then any solution

[4]We interpret x as the proportion of the consumer choosing B and y as the probability that supplier chooses to release cheap edition. So in our model, the interpretation of mixed strategy of each player is different.

$(x,y)(t)$ of this dynamic system passing through the interior of $[0,1]\times[0,1]$ converges to $(x^*,y^*)=\left(0,\dfrac{p_L-b+D}{p_L+D}\right)$.

(2)If $b>\dfrac{(p_L+D)[(d-p_L)-(u-p_H)]}{d-p_L}$, then this dynamic system has an asymptotically stable equilibrium point such that,

$$(x^*,y^*)=\left(\frac{(d-p_L)(p_L+D-b)-(u-p_L)(p_L+D)}{(d-p_L)(p_L-b)-(u-p_H)p_L},\frac{u-p_H}{d-p_L}\right).$$

The implication of this proposition is here. (1) means if supplier's choice is not biased(that is, $b=0$), then he tends always to release the cheap edition and consumers tend always to wait for release of it. We call this result the short-run equilibrium[5]. In contrast, (2) means if supplier's choice is sufficiently biased, then it is possible that a certain number of consumers buy immediately. Moreover, this equilibrium point converges to $\left(1,\dfrac{u-p_H}{d-p_L}\right)$ as $b\to+\infty$, and the payoff of supplier converges to $p_H-\dfrac{u-p_H}{d-p_L}D$. If D is sufficiently small, its payoff is larger than p_L, the payoff of short-run equilibrium. So we can conclude that if D is sufficiently small, supplier's biased decision may improve long-run payoff.

proof Suppose $(x,y)(t)$ is the solution and $(x,y)(t^*)$ is in the interior of $[0,1]\times[0,1]$. At first, we confirm $(x,y)(t)$ is extendable on $[t^*,+\infty[$ and $(x,y)(t)\in[0,1]\times[0,1]$ for every $t\geq t^*$.

Clearly, $(x,y)(t)$ is the solution of the dynamic system below;

$$\begin{aligned}\dot{x}&=x(1-x)[(u-p_H)-y(d-p_L)],\\ \dot{y}&=y[(1-y)[(1-x)p_L+D]-b(1-x)],\end{aligned}$$

where the right-hand side is defined on $\mathbb{R}^2$. As is well known, if $(x,y)(t)$ is not extendable on $[t^*,+\infty[$, there has to be $t\geq t^*$ such that $(x,y)(t)\notin[0,1]\times[0,1]$. Therefore, it suffices to show that $(x,y)(t)\in[0,1]\times[0,1]$ for all $t\geq t^*$.

Suppose $x(t)\leq 0$ for some $t>t^*$. Then we have $x(t')=0$ for some $t'\in[t^*,t]$ and thus $x\equiv 0$, implying $x(t^*)=0$, a contradiction. So we have $x(t)>0$ for all $t>t^*$. Similarly, we get $x(t)<1$ and $y(t)>0$ for all $t>t^*$. Next, suppose $y(t)>1$ for some $t>t^*$. Let $\bar{t}$ is the maximal element of the

[5]In fact, the point $(0,1)$ represents the only Nash equilibrium of this game.

nonempty set $\{t' \in [t^*, t] | y(t) = 1\}$. If $b > 0$, we have $\dot{y}(\bar{t}) = -b(1-x(\bar{t})) < 0$, so that, there exists $\hat{t} \in]\bar{t}, t[$ such that $y(\hat{t}) < 1$. Hence we have there exists $t' \in]\hat{t}, t[$ such that $y(t') = 1$, contradicting the definition of $\bar{t}$. If $b = 0$, we have $y \equiv 1$ and thus $y(t) = 1$, a contradiction. So we must have $(x, y)(t)$ is extendable on $[t^*, +\infty[$.

(1) We separates this situation into three cases; I) $b = 0$, II) $0 < b < \dfrac{(p_L + D)[(d-p_L) - (u-p_H)]}{d-p_L}$, III) $b = \dfrac{(p_L + D)[(d-p_L) - (u-p_H)]}{d-p_L}$.

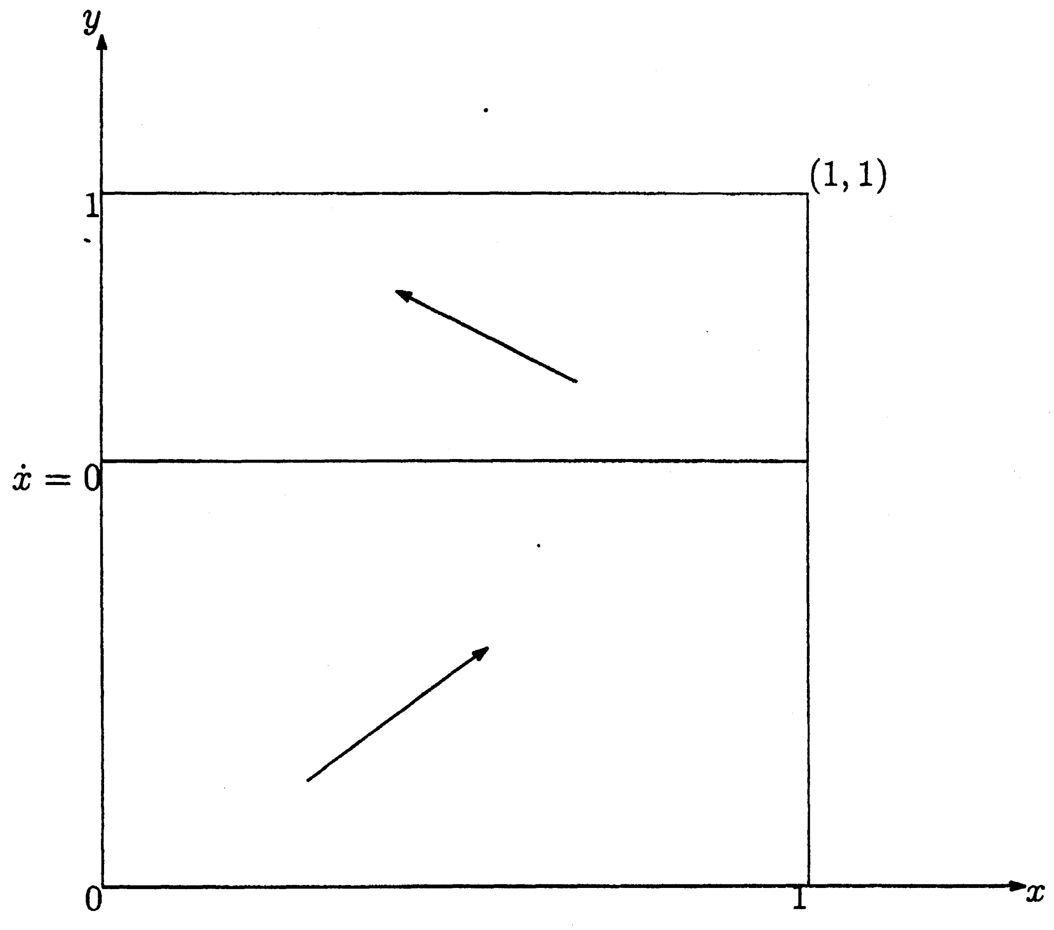

Figure 1: $b = 0$.

Case I: Assume $b = 0$. Then, we have $y(t)$ increases whenever $y(t) \in]0, 1[$, and thus $y(t)$ converges to $y^* = \sup y([t^*, +\infty[) \in [0, 1] \times [0, 1]$. If $y^* < 1$,

$$\dot{y}(t) = y(t)(1 - y(t))[(1 - x(t))p_L + D] \geq y(t^*)(1 - y^*)D > 0,$$

for all $t \geq t^*$, so that, $y(t) \to +\infty$ as $t \to \infty$, a contradiction. So $y^* = 1$. Therefore, we must have there exists $\bar{t} \geq t^*$ such that $0 < x(\bar{t}) < 1$ and $y(\bar{t}) > \frac{u-p_H}{d-p_L}$. Hence we have $\dot{x}(t) < 0$ for all $t \geq \bar{t}$ and thus $x(t)$ converges to

$x^* = \inf x([\bar{t}, +\infty[)$. If $x^* > 0$,

$$\begin{aligned}\dot{x}(t) &= x(t)(1-x(t))[(u-p_H) - y(t)(d-p_L)] \\ &\leq x^*(1-x(\bar{t}))[u-p_H-y(\bar{t})(d-p_L)] < 0,\end{aligned}$$

for all $t \geq \bar{t}$, so that, $x(t) \to -\infty$ as $t \to \infty$, a contradiction. So $x^* = 0$. This completes the proof of case I.

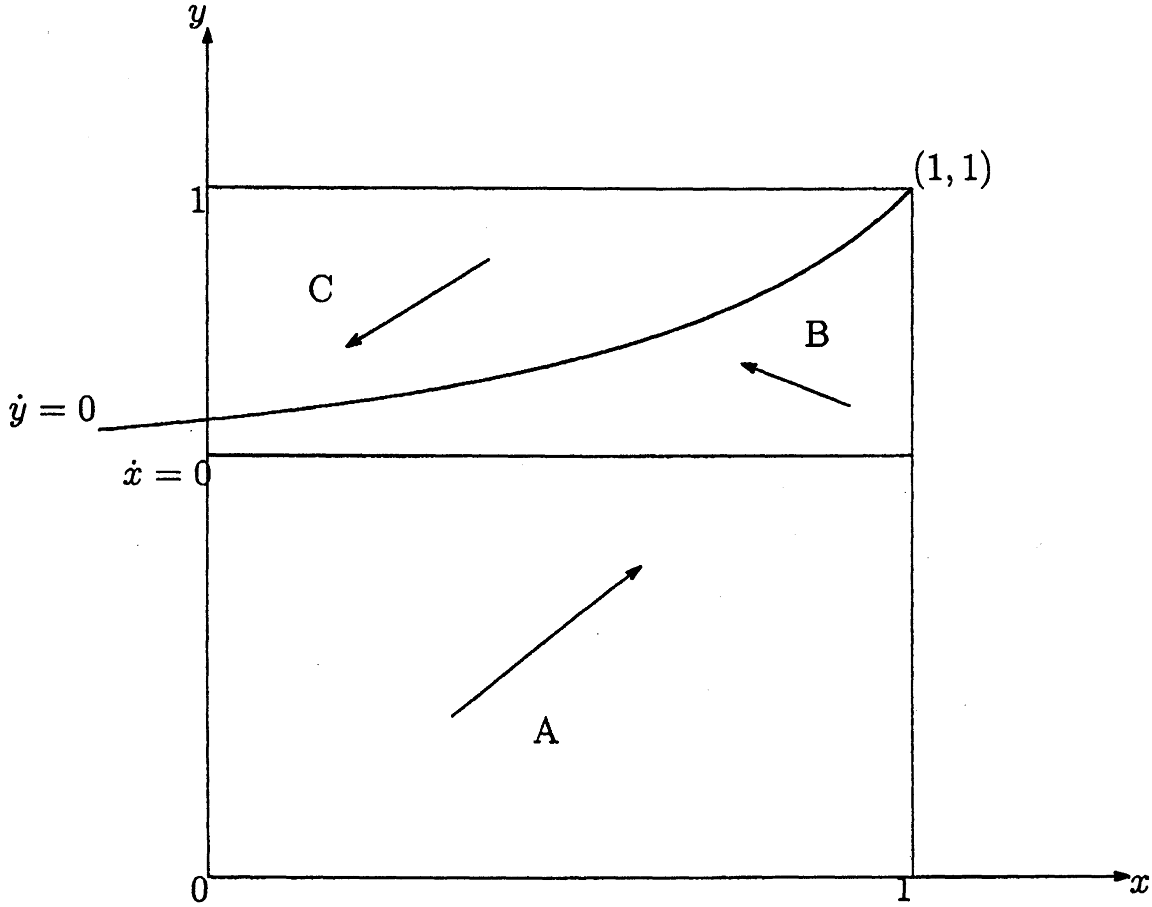

Figure 2: $0 < b < \frac{(p_L+D)((d-p_L)-(u-p_H))}{d-p_L}$.

Case II: Assume $0 < b < \dfrac{(p_L+D)[(d-p_L)-(u-p_H)]}{d-p_L}$.

See figure 2. We separates $[0,1] \times [0,1]$ into three areas. Area A is separated from area B by the $\dot{x} = 0$ line, and area B is separated from area C by the $\dot{y} = 0$ curve. We can easily check that

$$\dot{x} = 0 \Leftrightarrow y = \frac{u-p_H}{d-p_L},$$

$$\dot{y} = 0 \Leftrightarrow y = \frac{(1-x)(p_L-b)+D}{(1-x)p_L+D} \equiv g(x).$$

By easy computation, we have

$$g'(x) = \frac{bD}{((1-x)p_L + D)^2} > 0,$$

$$g''(x) = \frac{2bDp_L}{((1-x)p_L + D)^3} > 0,$$

whenever $x < 1$. Note that $g(0) = \dfrac{p_L - b + D}{p_L + D}$.

Now, we will show that every solution passing through area A must enter area B. In fact,

$$\begin{aligned}
\dot{y}(t) &\geq y(t^*)\left[\frac{(d-p_L)-(u-p_H)}{(d-p_L)}[(1-x(t))p_L + D] - b(1-x(t))\right] \\
&> y(t^*)x(t)D\frac{(d-p_L)-(u-p_H)}{d-p_L} \\
&\geq y(t^*)x(t^*)D\frac{(d-p_L)-(u-p_H)}{d-p_L} > 0,
\end{aligned}$$

whenever $y(t) \leq \dfrac{u-p_H}{d-p_L}$. So we must have $y(t) > \dfrac{u-p_H}{d-p_L}$ for some $t \geq t^*$.

Next, we will show that every solution passing through area B must not reenter area A. Suppose $(x,y)(t^*)$ is in area B and $(x,y)(t)$ is in area A for some $t > t^*$. Then $y(t^*) > \dfrac{u-p_H}{d-p_L} > y(t)$. Define $\bar{t} = \sup\{t' \in [t^*, t] | y(t') = \dfrac{u-p_H}{d-p_L}\}$. Then $\dot{y}(\bar{t} > 0$, implying $y(t') > \dfrac{u-p_H}{d-p_L}$ for some $t' \in [\bar{t}, t]$ and thus $y(t'') = \dfrac{u-p_H}{d-p_L}$ for some $t'' \in]\bar{t}, t]$, a contradiction.

Therefore, we have there exists $\bar{t}$ such that $\dot{x}(t) < 0$ for all $t \geq \bar{t}$ and thus $x(t)$ converges to $x^* = \inf x([\bar{t}, +\infty[)$.

Suppose $(x,y)(t)$ belongs to neither area C nor the curve $\{(x, g(x)) | x \in [0,1]\}$ for all $t \geq \bar{t}$. Then $\dot{y}(t) > 0$ for all $t \geq \bar{t}$ and thus $y(t)$ converges to $y^* = \sup y([\bar{t}, +\infty[)$. Suppose $x^* > 0$. Then,

$$\dot{x}(t) \leq x^*(1 - x(\bar{t}))[(u-p_H) - y(\bar{t})(d-p_L)] < 0,$$

for all $t \geq \bar{t}$. Hence we must have $x(t) \to -\infty$ as $t \to \infty$, a contradiction. So we have $x^* = 0$. Since $(x,y)(t)$ never belongs to area C for all $t \geq \bar{t}$, we must have $y^* \leq g(0)$. Suppose $y^* < g(0)$. Define

$$s^* = \inf\left\{(1-y)((1-x)p_L + D) - b(1-x) | x \in [0,1], y \in \left[\frac{u-p_H}{d-p_L}, y^*\right]\right\}.$$

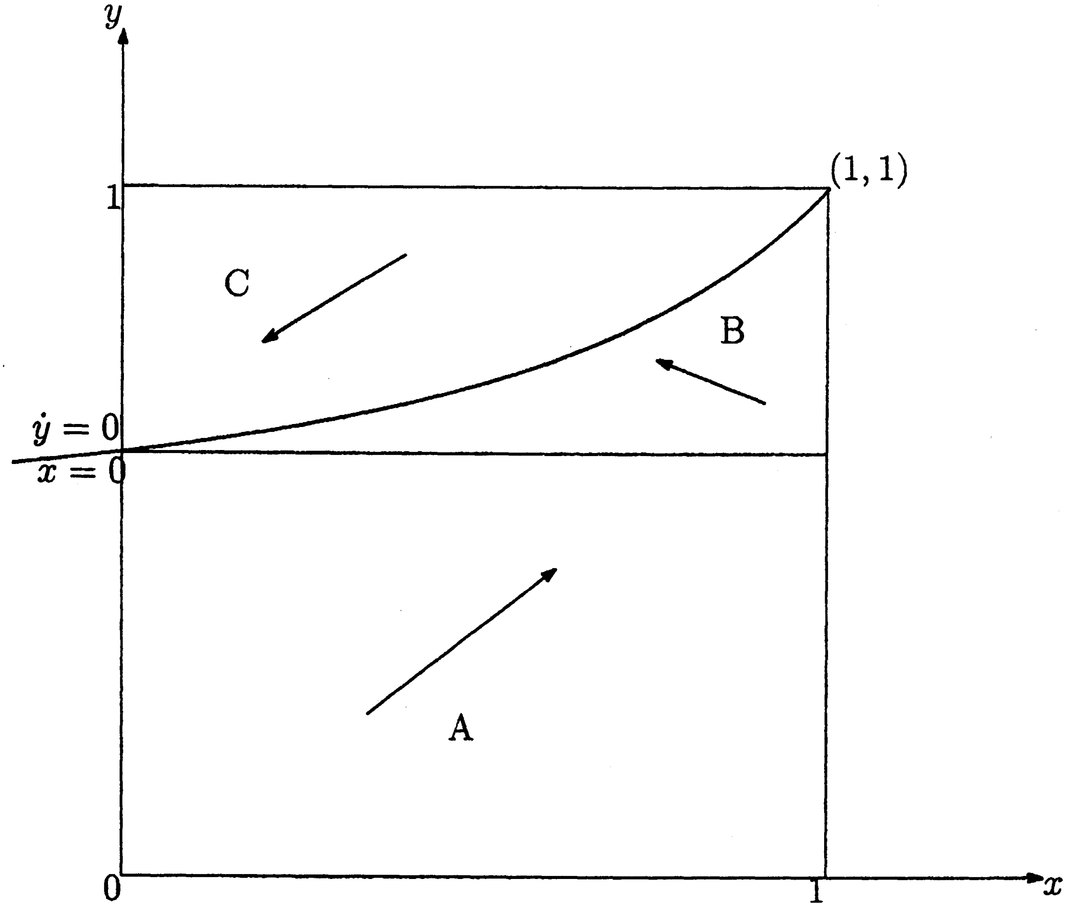

Figure 3: $b = \frac{(p_L+D)((d-p_L)-(u-p_H))}{d-p_L}$.

Since $y^* < g(0)$ and $g'(x) > 0$ for all $x < 1$, we must have $s^* > 0$ and thus $\dot{y}(t) \geq y(\bar{t})s^* > 0$ for all $t \geq \bar{t}$, implying $y(t) \to +\infty$ as $t \to \infty$, a contradiction.

Lastly, we treat the case there exists $\hat{t}$ such that $(x, y)(\hat{t})$ is in area C. We will show that $(x, y)(t)$ stays at area C whenever $t \geq \hat{t}$. Suppose on the contrary $(x, y)(t)$ is in area B for some $t \geq \hat{t}$. Let $\tilde{t}$ be the greatest element of $\{t' \in [\hat{t}, t] | \dot{y}(t') = 0\}$. Then we must have $\dot{x}(\tilde{t}) < 0$ and thus there exists $t' \in]\tilde{t}, t]$ such that $\dot{y}(t') < 0$. Since $\dot{y}(t) > 0$, there exists $t'' \in]\tilde{t}, t]$ such that $\dot{y}(t'') = 0$, contradicting the definition of $\tilde{t}$.

So we have $(x, y)(t)$ is in area C for all $t \geq \hat{t}$. Therefore, we must have $\dot{x}(t) < 0$ and $\dot{y}(t) < 0$ whenever $t \geq \hat{t}$, and thus $(x, y)(t) \to (x^*, y^*)$, where $x^* = \inf x([\hat{t}, +\infty[)$ and $y^* = \inf y([\hat{t}, +\infty[)$. We can easily check that $x^* = 0$ and $y^* \geq g(0)$. Suppose $y^* > g(0)$. Then there exists $\bar{t} \geq \hat{t}$ such that

$y^* \geq x(\tilde{t})$. Define

$$s^* = \sup\{(1-y)((1-x)p_L + D) - b(1-x) | x \in [0, x(\tilde{t})], y \in [y^*, 1]\}.$$

Then $s^* < 0$ and thus $\dot{y}(t) \leq y^* s^* < 0$ for all $t \geq \tilde{t}$, implying $y(t) \to -\infty$ as $t \to \infty$, a contradiction.

Case III: Assume $b = \dfrac{(p_L + D)[(d - p_L) - (u - p_H)]}{d - p_L}$.

The difference from case II is where the $\dot{y} = 0$ curve closses the $\dot{x} = 0$ line at $x = 0$. By the similar argument on case II, we can show that there exists $\hat{t} > t^*$ such that $(x, y)(\hat{t})$ is in area C. The rest of the proof is the same as the last part of the proof of case II.

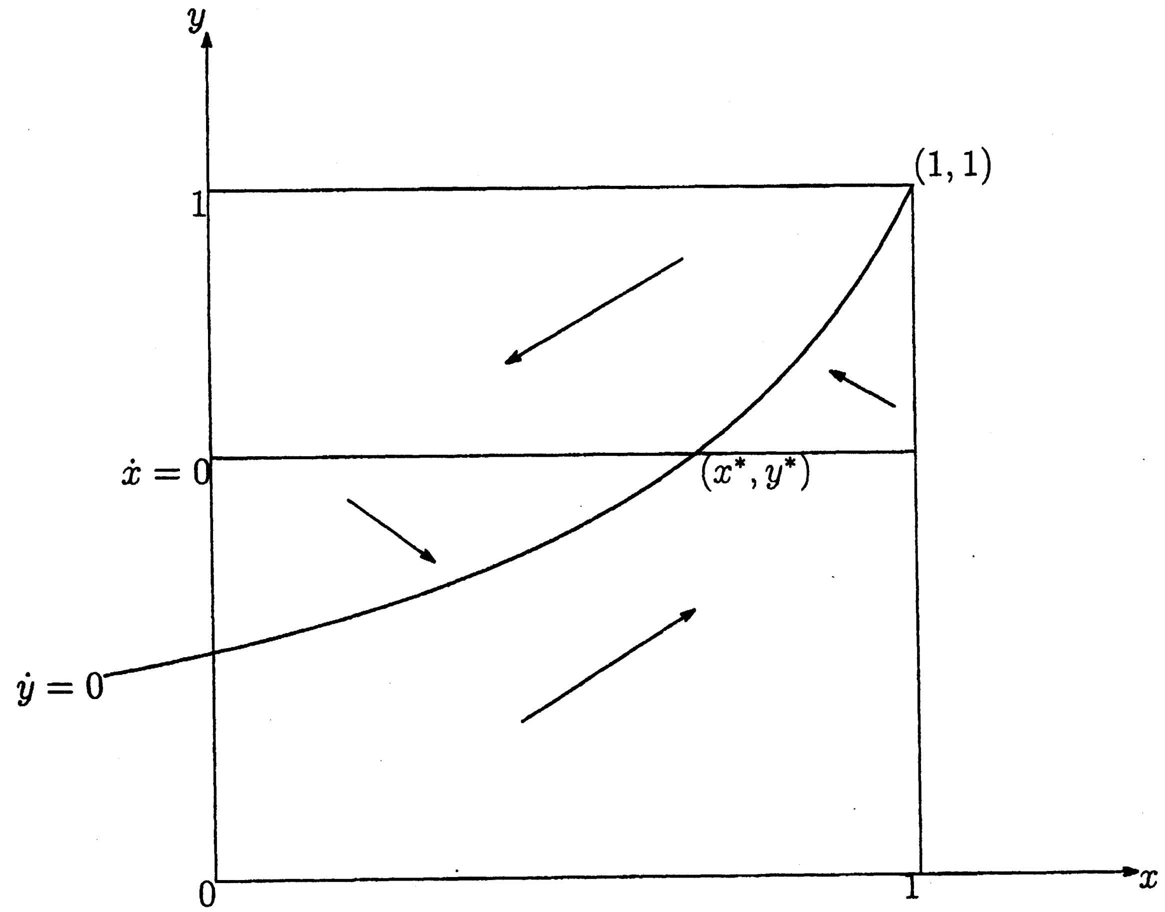

Figure 4: $b > \frac{(p_H+D)((d-p_L)-(u-p_H))}{d-p_L}$.

(2) Since (x^*, y^*) is a solution of $g(x) = \dfrac{u - p_H}{d - p_L}$, we must have $\dot{x} = \dot{y} = 0$ at (x^*, y^*). So it suffices to show that (x^*, y^*) is asymptotically stable. To do this, it suffices to show that for every eigenvalue λ of the Jacobian matrix, its

real part is negative. Since $\dot{x} = 0$ and $\dot{y} = 0$ at (x^*, y^*), its Jacobian matrix at (x^*, y^*) is

$$\begin{pmatrix} 0 & -x^*(1-x^*)(d-p_L) \\ y^*[b-(1-y^*)p_L] & -y^*((1-x^*)p_L + D) \end{pmatrix}$$

and thus, its characteristic equation is

$$\lambda^2 + y^*((1-x^*)p_L + D)\lambda + x^*(1-x^*)y^*(d-p_L)[b-(1-y^*)p_L] = 0.$$

Let λ_1, λ_2 be the solution of this equation. Then,

$$\lambda_1 + \lambda_2 = -y^*((1-x^*)p_L + D) < 0,$$
$$\lambda_1\lambda_2 = x^*(1-x^*)y^*(d-p_L)[b-(1-y^*)p_L].$$

Note that, if λ_1 is not a real number, then $\bar{\lambda}_1 = \lambda_2$ and $\lambda_1 + \lambda_2 < 0$, so the real part of λ_1, λ_2 must be less than 0. So we can suppose $\lambda_1, \lambda_2 \in \mathbb{R}$ without loss of generality. Then,

$$b - (1-y^*)p_L > \frac{D[(d-p_L) - (u-p_H)]}{d-p_L} > 0,$$

implying $\lambda_1, \lambda_2 < 0$. It completes the proof. ■

3 Concluding remarks

We try to explain the strange behavior for suppliers of books or softwares which release the cheap edition of their products only when this product has a good sale, and show that if supplier's short-run decision is biased, then his long-run payoff may improve. So suppliers need not be irrational, and his strange behavior may improve his payoff.

To conclude above, we used a dynamic system similar to the replicator dynamics of a strategic form game represents this situation, except the supplier's behavior is biased. Compared to alternative explanations(especially, using the folk theorem), this explanation has some virtues. First, this explanation need not require the clever action of consumers. Second, supplier's decision process to realize the long-run equilibrium emerges specifically, and matches actual behavior. So this explanation is better than the others.

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